

– Module 12 –

# Fundamental Statistical Tools III

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# ASSUMPTION CHECK

- Most statistical tests (especially **parametric** tests) make a number of **assumptions**
- **EXAMPLE: Student's t-test assumptions**
  - Data values are **continuous**
  - Data is (approximately) **normally distributed**
  - Data points are **independent**
  - Samples have similar amount of variance  
i.e. **homogeneity of variance**

## Shapiro-Wilk normality test

```
data: bci.wakhan  
W = 0.98726, p-value = 0.7695
```

## Shapiro-Wilk normality test

```
data: bci.altai  
W = 0.98112, p-value = 0.718
```

## Bartlett test of homogeneity of variances

```
data: BCI_2 by Location  
Bartlett's K-squared = 2.3336, df = 1, p-value = 0.1266
```

### ## METHOD 1: graphical inspection

#### # histograms to check normality

```
hist(bci.wakhan)
```

```
hist(bci.altai)
```

#### # boxplot to check homogeneity of variance

```
boxplot(BCI_2 ~ Location, data = uncia)
```

### ## METHOD 2: statistical tests

#### # Shapiro-Wilk test to check normality

```
shapiro.test(bci.wakhan)
```

```
shapiro.test(bci.altai)
```

#### # Bartlett's test to check homogeneity of variance

```
bartlett.test(BCI_2 ~ Location, data = uncia)
```

# P-VALUE

- **Two sample t-test** to compare two sample means

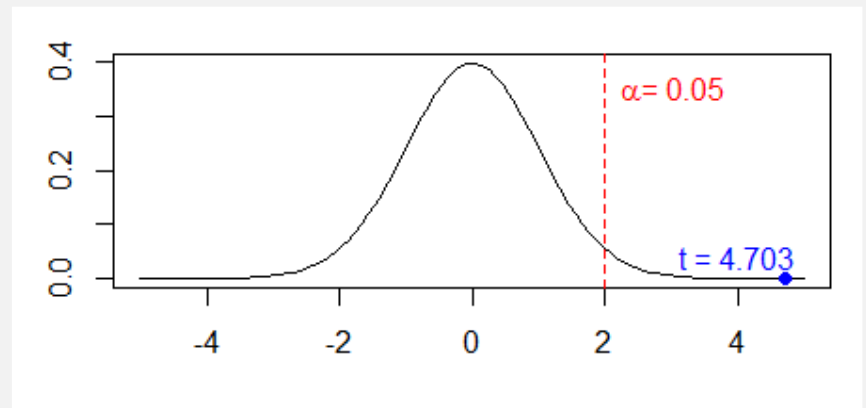
$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{(s^2(\frac{1}{n_1} + \frac{1}{n_2}))}}$$

- Result: there is a **significant difference** between the two group means at the significance level  $\alpha = 0.05$

```
# t-test to compare mean BCI_2 between the two locations
bci.wakhan <- subset(uncia$BCI_2, uncia$Location == "Wakhan")
bci.altai <- subset(uncia$BCI_2, uncia$Location == "Altai")
t.test(x = bci.wakhan, y = bci.altai, alternative = "two.sided", var.equal = TRUE)
```

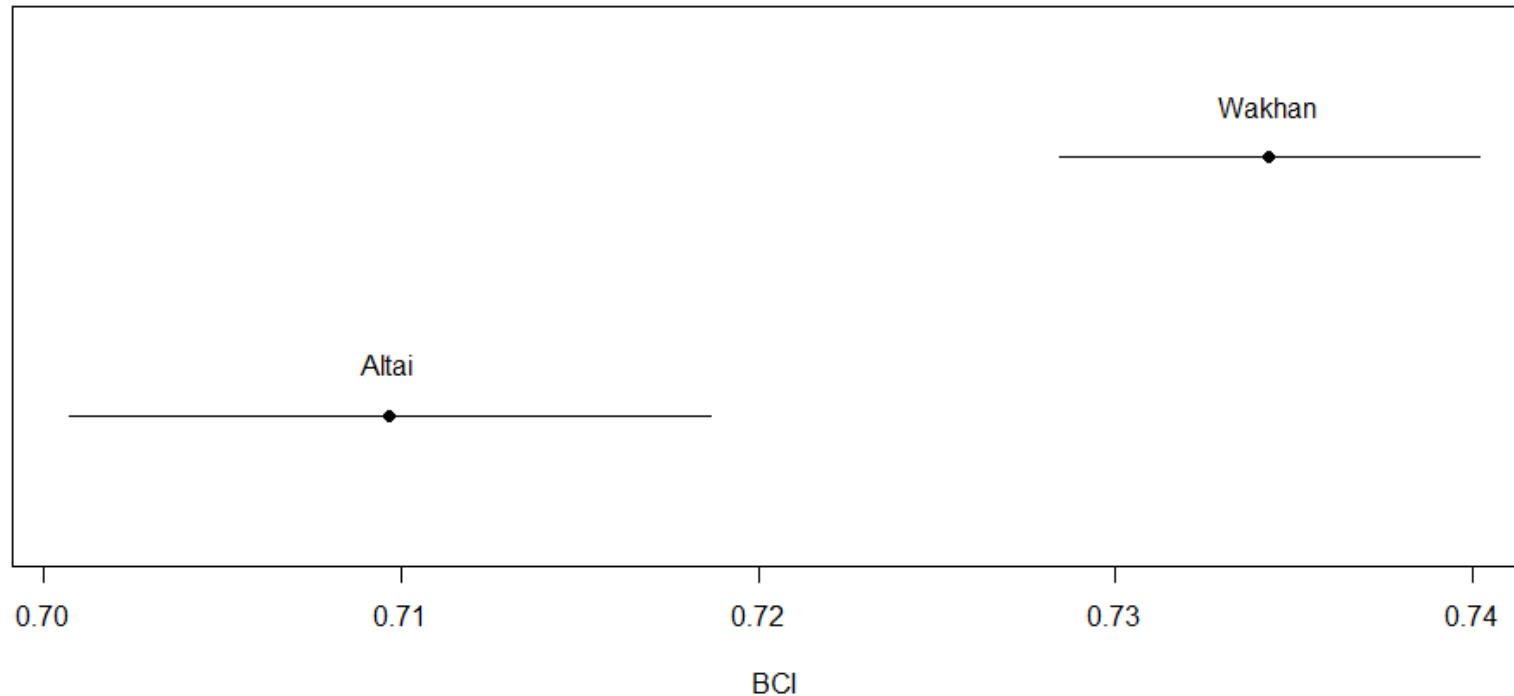
## Two Sample t-test

```
data: bci.wakhan and bci.altai
t = 4.7027, df = 101, p-value = 8.155e-06
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 0.01422979 0.03499355
sample estimates:
mean of x mean of y
0.7342995 0.7096879
```



# CONFIDENCE INTERVALS

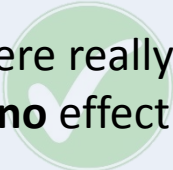
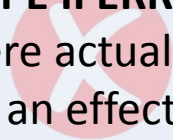
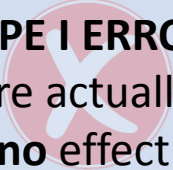
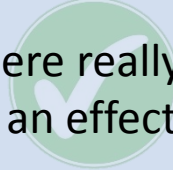
- T-test: the two sample means (BCI) are **significantly different** at  $\alpha = 0.05$
- There is **no overlap** between the 95% confidence intervals for the two estimates of population means



# TYPE I & TYPE II ERRORS

- **Type I error** = rejection of a true null hypothesis
- **Type II error** = non-rejection of a false null hypothesis

- **How to avoid ...**
  - increase sample size
  - think carefully about study design
  - set a different  $\alpha$   
e.g. 0.01 or 0.1
  - make sure that no key assumptions have been violated
  - choose a better suited analytical framework

		<i>H<sub>0</sub></i> What the null hypothesis really is	
		TRUE	FALSE
<i>H<sub>0</sub></i> What we think the null hypothesis is	TRUE	there really is <b>no effect</b> 	<b>TYPE II ERROR</b> there actually is an effect 
	FALSE	<b>TYPE I ERROR</b> there actually is <b>no effect</b> 	there really is an effect 

# ONE-WAY ANOVA

- **Analysis of variance**
- To compare more than two sample means

- **Variance:**  $\frac{\sum (x - \bar{x})^2}{n-1}$  ← Sum of squares (SST)

- SSW:  $\sum (x - \bar{x})^2$

- SSB:  $n \sum (\bar{x} - \bar{\bar{x}})^2$

- **SST = SSW + SSB**

## F statistic

$$\frac{MSB}{MSW} = \frac{SSB/(c-1)}{SSW/(n-c)} \begin{array}{l} \leftarrow \text{Degrees of} \\ \leftarrow \text{freedom} \end{array}$$

- MSB: mean square between groups
- MSW: mean square within groups
- c: number of categories
- n: number of samples

# ONE-WAY ANOVA

$H_0: \mu_1 = \mu_2 = \mu_3$  : all means are equal

$H_1$  (**alternative hypothesis**): at least one population mean is different from the rest

## Assumptions:

- Population is **normally distributed**
- Measurement is interval or ratio
- Categories are **independent**
- Samples are **independent**
- **Homogeneity** of variance

# PAIRWISE T-TESTS

- Used to figure out **pair-wise differences** between groups  
i.e. **post-hoc analysis**
- The problem with multiple comparisons: false positives!
- **Bonferroni** correction:  $\alpha/m$
- **Holm** method/**Holm-Bonferroni** method: arrange significant results in ascending order of p-value and check significance against modified p-values:

$$P_k < \frac{\alpha}{m + 1 - k}$$



# TWO-WAY ANOVA

- Used to test effect of **two grouping variables** simultaneously, as well as their interaction
- **Post-hoc analysis: Tukey's** honestly significant difference test

	Altai	Wakhan
<b>Females</b>	88	98
	90	99
	91	101
	92	103
<b>Males</b>	110	112
	111	116
	112	118
	113	119

```
# calculating anova
mod1 <- aov(Weight.kg ~ Location*Sex, data = uncia)
summary(mod1)

# Post-Hoc analysis
TukeyHSD(mod1, conf.level=.95)

# Plot Tukey
par(mar=c(4.1, 13, 4.1, 2.1))
plot(TukeyHSD(mod1, conf.level=.95), las = 2)
```